LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F7035T
Examination date	2014-08-30
Time	09.00 - 14.00

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **van der Waals gas**

   The partition function Z for a gas of N interacting particles is given by

   $$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

   where a and b are constants and V is the volume. Derive the equation of state of the gas and also evaluate it's energy U.

   (3p)

2. **Rotation of a di–atomic molecule**

   The kinetic energy of a di–atomic molecule consists of a translational part and a rotational part. The rotational energy $\epsilon(j)$ has quantised levels and for a di–atomic molecule these are given by:

   $$\epsilon(j) = j(j+1)\epsilon_0$$

   where j is an integer with the following values $j = 0, 1, 2, ...$ The degeneracy $g(j)$ of each level is given by:

   $$g(j) = 2j + 1.$$

   a) Calculate the partition function for the rotational degrees of freedom $Z_R(\tau)$.

   b) Approximate $Z_R(\tau)$ in the limit $\tau >> \epsilon_0$ by an integral and calculate the specific heat C_v in this limit.

   c) Approximate $Z_R(\tau)$ in the limit $\tau << \epsilon_0$ by truncating the sum to two terms and calculate the specific heat C_v in this limit.

   d) Draw a figure showing the results from b) and c)

   (3p)

TURN PAGE!

3. **The three dimensional Ising–model in the mean field approximation**

   The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

$$H = -J \sum_{<i,j>} s_i s_j ,$$

   where the classical spins s have the following states $+1$ and -1. The spins s_i interact with their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state, ie the magnetisation at temperature $\tau = 0$ is $< m >= \frac{1}{L^3} \sum_i s_i = 1$.

   As the temperature is raised the magnetisation disappears at a specific temperature the Curie temperature τ_c.

   As the temperature approaches τ_c from below the magnetization goes to zero according to $m \propto (\tau_c - \tau)^{\beta}$. Within the mean field approximation calculate the exponent β for the magentisation.
   ($\tanh(x) \approx x - x^3/3$ for small x).

   (3p)

4. **Harmonic oscillator**

   A three dimensional harmonic oscillator has energy levels

$$\epsilon_{n_1,n_2,n_3} = (n_1 + n_2 + n_3 + \frac{3}{2})\ \hbar\omega$$

   where n_1, n_2, n_3 är are integers from 0 to ∞.

   a) At what temperature is the probability for the oscillator to be in a state of energy $\frac{3}{2}\hbar\omega$ or $\frac{5}{2}\hbar\omega$ the same?

   b) How large is this probability ?

   (3p)

5. **The triple point of ammonia**

   In the vicinity of the triple point, the vapor pressures of solid and liquid ammonia are respectively given by p_s (in N/m^2) $\ln p_s = 27.923 - 3754/T$ and $\ln p_l = 24.383 - 3063/T$. The temperature T is in Kelvin.

   Hint: Assume the the ideal gas law applies to the gas phase. The volume of molecules in the solid phase may be neglected.

   a) At what temperature is the triple point?

   b) What are the latent heats of sublimation (solid-gas) and of vaporisation (liquid-gas) at the triple point?

   c) What is the latent heat of melting (solid-liquid) at the triple point?

   (3p)

GOOD LUCK !