LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F7035T
Examination date	2016-03-23
Time	09.00 - 14.00

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
Teacher on duty: Hans Weber   Tel: (49)2088, Room E304
Examiner: Hans Weber   Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **The three dimensional Ising–model in the mean field approximation**

   The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

   $$H = -J \sum_{<i,j>} s_i s_j,$$

   where the classical spins s have the following states $+1$ and -1. The spins s_i interact with their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state, ie the magnetisation at temperature $\tau = 0$ is $< m >= \frac{1}{L^3} \sum_i s_i = 1$.

   As the temperature is raised the magnetisation disappears at a specific temperature the Curie temperature τ_c. As the temperature approaches τ_c from below the magnetization goes to zero according to $m \propto (\tau_c - \tau)^\beta$.

   Within the mean field approximation calculate the exponent β for the magentisation.

   ($\tanh(x) \approx x - x^3/3$ for small x). (3p)

2. **Pressure and energy of a gas of Photons**

   In this problem you are asked to derive an expression for the pressure of a photon gas confined to a cube with the side L.

   a) Start by deriving an expression for the energy density $\frac{U}{V}$ (Stefan–Boltzmanns T^4 law). (Hint use the Planck distribution and that the relation between the frequency of the photon ω_n and the mode $n = \sqrt{n_x^2 + n_y^2 + n_z^2}$ is given by $\omega_n = n\pi c/L$.)

   b) Derive the expression for the pressure p of the photon gas.
   (Hint use the entropy σ.)

   c) The energy of a mono atomic gas is given by $pV = \frac{2}{3}U$. Derive the corresponding expression for the photon gas.

   (3p)

3. **Velocity of Fermions**

A piece of metal of volume V contains N conduction electrons each of mass m. Calculate (for zero temperature) the averages $\langle v \rangle$ and $\langle v^2 \rangle$, where v is the absolute value of the velocity of the electron.

The conduction electrons may be treated as a free electron gas. Present your answer in terms of the Fermi velocity v_F.

(3p)

4. **Photons**

At a blast of an atomic bomb temperatures in the range of 10^6K can be reached. Assume this is true for a 'fire' ball of diameter $d = 10$cm. Evaluate the following:

a) The total emitted power from this 'fire' ball.

b) What is the radiation flux at a distance of 2.0 km?

c) At what wavelength λ peaks the output of power?

(3p)

5. **Gas-solid equilibrium**

In a container of volume V a substance solid phase is in equilibrium with its gas phase. The atoms have a binding energy $-\epsilon_0$ to the solid phase.

Use the following approximations. The substance is mono atomic and for the gas phase the ideal gas applies. Further the gas phase has volume V independent of the amount in the solid phase. Also the entropy of the solid phase is negligible.

Let the total number of atoms be $N = N_s + N_g$ where N_s and N_g are the number of atoms in the solid phase and gas phase.

a) Express the free energy of the system F. (Hint $F = F_s + F_g$.)

b) Minimize F with respect to N_g and derive an expression for N_g.

c) Estimate ϵ_0 for H_2O using the data in the table. Answer in electron volts ! (Assume the ideal gas applies to the gas phase of H_2O. Hint: Note that the range of temperatures is small if you need to make approximations for to the expression of p_g.)

T (^oC)	saturation pressure (kPa)
-2	0.5176
-6	0.3689
-10	0.2602
-14	0.1815
-20	0.1035
-30	0.0381
-40	0.0129

(3p)

GOOD LUCK !