LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	F7035T
Examination date	2017-03-24
Time	09.00 - 14.00

Examination in: Statistical Physics and Thermodynamics
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, Collection of formulae

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **Phase separation in a mixture of two liquids**

   Below you see two figures, I and II. They show the average energy u of an atom as a function of the mixing fraction $x = N_B/(N_A + N_B)$, where N_A and N_B are the number of atoms of kind A and B.

   Which one of the figures I or II will support a separation of the homogeneous mixture into two phases as the temperature is lowered. Where one phase is rich of atoms of kind A and one is rich of atoms of kind B. Explain and motivate why. (Hint, Support your arguments with figures, you may neglect the term pV in the Gibbs free energy and support your arguments on Helmholtz free energy) (3p)

   fig. I

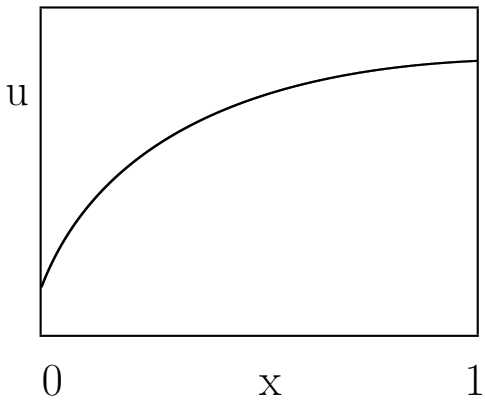
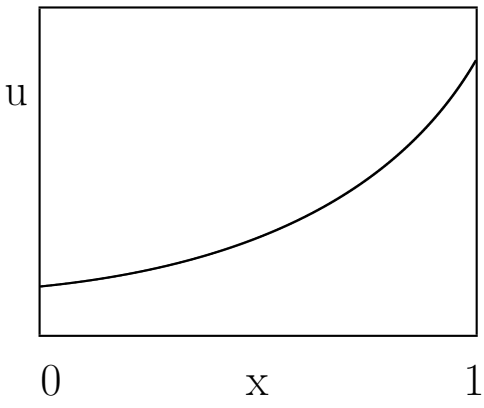


   fig. II



2. **Two dimensional ideal Fermi gas**

   A two dimensional Fermi gas can be realised in semiconductors or thin ^{3}He films

   For an ideal Fermi gas in two dimensions derive the density of states $D(\epsilon)$.

   (3p)

TURN PAGE!

3. **Harmonic oscillator**

A two dimensional harmonic oscillator has energy levels according to

$$\epsilon_{n_1,n_2} = (n_1 + n_2 + 1)\ \hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1, 2, 3, ...\infty$. The oscillator is coupled to a heatbath of temperature τ with which the oscillator can exchange energy.

(a) Calculate the partition function of the oscillator for any temperature.

(b) At what temperature equals the probability to find the oscillator in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

(c) How large is this probability ?

(3p)

4. **van der Waals gas**

The partition function Z for a gas of N interacting particles is given by

$$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

where a and b are constants and V is the volume. Derive the equation of state of the gas and also evaluate it's energy U.

(3p)

5. **Interstitial atoms**

The atoms in a crystal of a monoatomic substance can be assumed to sit in either their original lattice positions or in so called interstitial positions. Atoms sitting at a interstitial position have a higher energy compared to if they had been at an ordinary site. The difference in energy is denoted by ϵ. The crystal has N atoms, N lattice sites and N interstitial positions. At a temperature τ, n interstitial sites are occupied by atoms.
Calculate the fraction n/N if $\tau << \epsilon$ and N and $n >> 1$.
(use the approximation $\ln n! = n \ln n - n$)

(3p)

GOOD LUCK !