LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

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| Course code | F7035T |
| Examination date | 2018-03-23 |
| Time | 09.00 - 14.00 |

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

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Allowed aids: Fysikalia/Fysika, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

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Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Entropy**

   Use simple arguments concerning the specific heat $C_v$ to answer the following questions.

   **a)** For a metal the temperature is changed from $300K$ to $800K$. By how large a factor will the entropy change for the conduction electrons?

   **b)** For the electromagnetic radiation inside a cavity the temperature is changed from $500K$ to $1200K$. By how large a factor will the entropy change for the radiation field inside the cavity?

   (3p)

2. **Schottky anomaly**

   A system consisting of one particle is attached to a thermal reservoir at temperature $\tau$. The particle can be in one of two states with energy 0 and $\epsilon_0$.

   **a)** What is the energy of the system $U \equiv \langle \epsilon \rangle$?

   **b)** Calculate the heat capacity at constant volume of the system $C_v(\tau)$.

   **c)** Is there a maximum in $C_v$ as a function of temperature, and if at what temperature $\tau_m$?

   (3p)

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3. **Rotation of a di–atomic molecule**

The kinetic energy of a di–atomic molecule consists of a translational part and a rotational part. The rotational energy $\epsilon(j)$ has quantised levels and for a di–atomic molecule these are given by:

$$\epsilon(j) = j(j+1)\epsilon_0$$

where $j$ is an integer with the following values $j = 0, 1, 2, ...$ The degeneracy $g(j)$ of each level is given by:

$$g(j) = 2j + 1.$$

a) Calculate the partition function for the rotational degrees of freedom $Z_R(\tau)$.

b) Approximate $Z_R(\tau)$ in the limit $\tau >> \epsilon_0$ by an integral and calculate the specific heat $C_v$ in this limit.

c) Approximate $Z_R(\tau)$ in the limit $\tau << \epsilon_0$ by truncating the sum to two terms and calculate the specific heat $C_v$ in this limit.

d) Draw a figure showing the results from b) and c)

(3p)

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4. **The DNA – molecule**

The DNA–molecule consists, as we know of two chains (see figure 1) which are held together by bonds between the base pairs adenine and thymine (AT - bonds) and cytosine and guanine (CG - bonds). This means that the DNA – molecule can function much like a zipper and opened by the bindings sequentially broken from one end or from both ends. For a DNA – molecule containing $N$ bonds, the energy cost to break a bond is $\epsilon$, and a broken bond can have $g$ different orientations.

**a)** Show that the partition sum for a DNA–zipper that only can be opened from one of its ends is given by:

$$Z^{(N)} = \sum_{N_b=0}^{N} g^{N_b} e^{-N_b \epsilon/\tau},$$

where $N_b$ is the number of broken bonds.

**b)** Evaluate explicitly and show that the fraction of broken bonds as $N \to \infty$ is 0 if $ge^{-\epsilon/\tau} < 1$ and 1 if $ge^{-\epsilon/\tau} > 1$.

(3p)

This is an example of a *first–order phase transition* of the same kind as when water changes from a liquid to a gas. The phase transition occurs at the temperature $T_c = \frac{\epsilon}{k_B \ln(g)}$. The phase transition from a closed to open an DNA–zipper and vice versa occurs not because of body temperature, but because the bonding energy $\epsilon$ and hence $T_c$ is changed due to catalytic influence on $\epsilon$, which gives the same result.

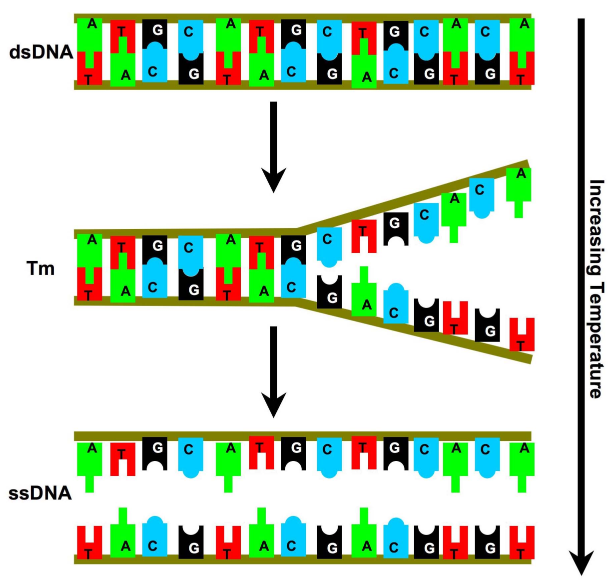


Figure 1: How DNA melts, taken from http://www.patentlens.net/daisy/RiceGenome/3663/3612.html.

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5. **Maxwell velocity distribution**

An experiment is designed to measure the Maxwell velocity distribution for a gas of Sodium (Na) atoms at $T = 280^o C$. The experimental set up is according to figure 2.

In the oven there is a gas of Sodium atoms. At the slit at A the atoms are allowed to leave the oven. The slit at A is usually closed by the rotating drum. But as the slit D in the drum and the slit A line up Sodium atoms are allowed to exit the oven into the rotating drum.

The drum has a diameter $d = 10.0$cm and rotates with an angular velocity $\omega$ around the axis C.

Determine the angular velocity $\omega$ required so that Sodium atoms in the beam travelling at the most probable velocity $v_{mp}$ in the beam will hit the slit D again as they have travelled across the drum. (ie the drum has rotated half a turn) (3p)

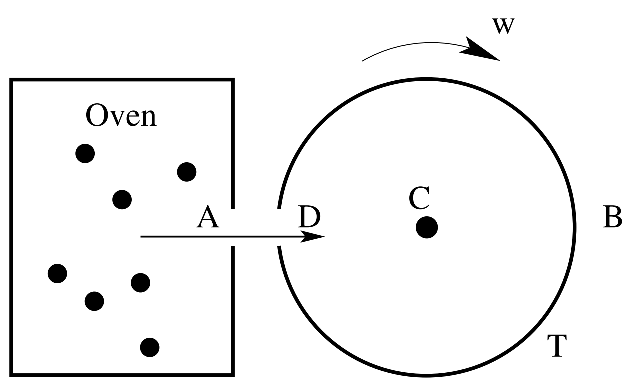


Figure 2: A principal experimental setup to determine the Maxwell velocity distribution.

GOOD LUCK !