Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
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Allowed aids: Fysikalia/Fysika, Physics Handbook, Beta, calculator, COLLECTION OF
FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that
they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the
examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **The three dimensional Ising–model in the mean field approximation**

   The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

   $$H = -J \sum_{<i,j>} s_i s_j,$$

   where the classical spins s have the following states $+1$ and -1. The spins s_i interact with
   their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state,
   ie the magnetisation at temperature $\tau = 0$ is $< m > = \frac{1}{L^3} \sum_i s_i = 1$.

   As the temperature is raised the magnetisation disappears at a specific temperature the
   Curie temperature τ_c. As the temperature approaches τ_c from below the magnetization goes
   to zero according to $m \propto (\tau_c - \tau)^\beta$.

   Within the mean field approximation calculate the exponent β for the magentisation.

   ($\tanh(x) \approx x - x^3/3$ for small x).   (3p)

2. **van der Waals gas**

   The partition function Z for a gas of N interacting particles is given by

   $$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_B T}}$$

   where a and b are constants and V is the volume. Derive the equation of state of the gas
   and also evaluate it's energy U.

                                                                                      (3p)

3. **Maxwell velocity distribution**

An experiment is designed to measure the Maxwell velocity distribution for a gas of Sodium (Na) atoms at $T = 300^oC$. The experimental set up is according to figure 1.

In the oven there is a gas of Sodium atoms. At the slit at A the atoms are allowed to leave the oven. The slit at A is usually closed by the rotating drum. But as the slit D in the drum and the slit A line up Sodium atoms are allowed to exit the oven into the rotating drum.

The drum has a diameter $d = 10.0$cm and rotates with an angular velocity ω around the axis C.

Determine the angular velocity ω required so that Sodium atoms in the beam travelling at the most probable velocity v_{mp} in the beam will hit the slit D again as they have travelled across the drum. (ie the drum has rotated half a turn) (3p)
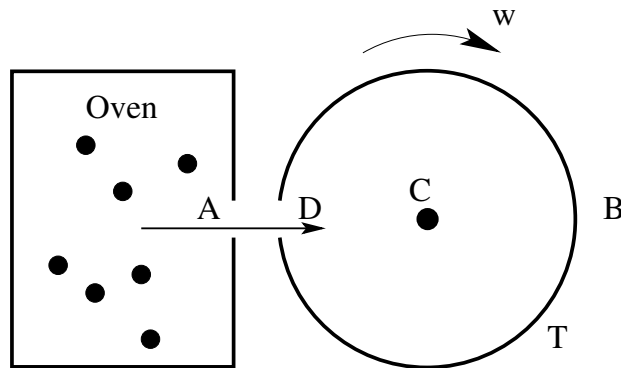

Figure 1: A principal experimental setup to determine the Maxwell velocity distribution.

4. **The triple point of ammonia**

In the vicinity of the triple point, the vapor pressures of solid and liquid ammonia are respectively given by p_s (in N/m^2) $\ln p_s = 27.923 - 3754/T$ and $\ln p_l = 24.383 - 3063/T$. The temperature T is in Kelvin.

Hint: Assume the the ideal gas law applies to the gas phase. The volume of molecules in the solid phase may be neglected.

a) At what temperature is the triple point?

b) What are the latent heats of sublimation (solid-gas) and of vaporisation (liquid-gas) at the triple point?

c) What is the latent heat of melting (solid-liquid) at the triple point?

(3p)

5. **Three questions**

1. What would be the greatest effect on the ideal gas law if there is a slight attractive force between the molecules?

   A. At low densities, the pressure would be less than that predicted by the ideal gas law.

   B. At high densities, the pressure would be less than that predicted by the ideal gas law.

   C. At high densities, the pressure would be greater than that predicted by the ideal gas law.

   D. At low densities, the pressure would be higher than that predicted by the ideal gas law.

   E. There is no effect.

2. Which has higher entropy - a mole of ideal gas at 20 ^{o}C occupying 10 liters or a mole of the same gas at 20 ^{o}C occupying 100 liters?

   A. The gas in 10 liters

   B. The gas in 100 liters

   C. No difference

   D. The statement contains not enough information.

3. How much higher entropy does a mole of ideal gas at 20 ^{o}C occupying 100 liters have compared to a mole of gas at 20 ^{o}C occupying 10 liters?

(3p)

GOOD LUCK !