LULEÅ UNIVERSITY OF TECHNOLOGY
Applied Physics

Course code	F7035T
Examination date	2019-05-29
Time	09.00 - 14.00

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Teacher on duty: Hans Weber Tel: (49)2088, Room E163
Examiner: Hans Weber Tel: (49)2088, Room E163

Allowed aids: Fysikalia/Fysika, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow.
Total number of problems: **5**. Maximum number of point is 15 p. 7.5 points is required to pass the examination. Grades 3: 7.5, 4: 10.0, 5: 12.0

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1. **Helium** ^3He

   Helium ^3He has spin $= \frac{1}{2}$ and may at low temperatures to a good approximation be described as an ideal Fermi gas. At these low temperatures ^3He is in the liquid phase with a density of $\rho = 83$ kg m^{-3}.

   a) Determine the Fermi temperature T_F and also the specific heat C_v of ^3He at T=0.2 K.

   b) Can you still use the approximations you did in a) if the temperature where say 2-3 K? If not why? If yes why?

   (3p)

2. **Adsorption**

   Consider an ideal gas (temperature T, chemical potential μ) in contact with a surface with N adsorption sites. Each adsorption site may be occupied by 0, 1 or 2 gas molecules. The energy of a vacant site is zero, the energy with one adsorbed molecule is $-\epsilon$ and the energy with two adsorbed molecules is $-\frac{3}{2}\epsilon$. ϵ can be positive or negative. There is no interaction between molecules at different adsorption sites.

   a) Calculate the grand canonical partition function for a fixed number N of adsorption sites.

   b) Use the grand canonical partition function to derive the mean number of adsorbed particles per site $< n >$ and the mean internal energy per site $< u >$ as a function of temperature T, μ and ϵ.

   (3p)

TURN PAGE!

3. **The three dimensional Ising–model in the mean field approximation**

The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

$$H = -J \sum_{<i,j>} s_i s_j,$$

where the classical spins s have the following states $+1$ and -1. The spins s_i interact with their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state, ie the magnetisation at temperature $\tau = 0$ is $< m >= \frac{1}{L^3} \sum_i s_i = 1$.

As the temperature is raised the magnetisation disappears at a specific temperature the Curie temperature τ_c. As the temperature approaches τ_c from below the magnetization goes to zero according to $m \propto (\tau_c - \tau)^\beta$.

Within the mean field approximation calculate the exponent β for the magentisation.

($\tanh(x) \approx x - x^3/3$ for small x). (3p)

4. **Harmonic oscillator**

A three dimensional harmonic oscillator has energy levels

$$\epsilon_{n_1,n_2,n_3} = (n_1 + n_2 + n_3 + \frac{3}{2})\ \hbar\omega$$

where n_1, n_2, n_3 är are integers from 0 to ∞.

a) At what temperature is the probability for the oscillator to be in a state of energy $\frac{3}{2}\hbar\omega$ or $\frac{5}{2}\hbar\omega$ the same?

b) How large is this probability ?

(3p)

5. **Diatomic molecule**

An ideal gas consists of N identical molecules. Each molecule consists of two atoms with the following rotational energy levels: $E(j) = j(j+1)\frac{\hbar^2}{2I}$, $j = 0, 1, 2, ...$ Where I is the moment of inertia. Each level is $(2j+1)$ times degenerate. For low temperatures determine to lowest order in temperature the contribution to C_v from the rotational degrees of freedom.

(3p)

GOOD LUCK !